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Influence of Van Hiele's Model on the Development of Mathematical Thinking in Longitude and Latitude among Secondary School Students in Sokoto State, Nigeria

Aliyu Bello Shehu[1]
Email: saliyutureta77@gmail.com
Tel: 07034514761

Hassan Muhammad Nasir[1]
Email: mohammad.nasiruhassan@ssu.edu.ng
Tel: 08067788633

Ibrahim Nasiru Tambuwal[1]
Email: nasiru.ibrahimtambuwal@ssu.edu.ng
Tel: 08036084320

Maccido Isah[1]
Email: isahmaccido20@gmail.com
Tel: 07037106344

[1]Department of Science Education, Sokoto State University, Sokoto

Abstract
This research aims at examining students' mathematical thinking ability. In order to achieve these objectives two research questions with their equivalence null hypotheses were generated to guide the study. A quasi-experimental design entailing pretest and posttest was employed. A sample of 267 (155 Male and 112 Female) students were selected using simple random sampling techniques through balloting method out of 28,897students during 2022/203 session. Mathematical Thinking Ability Test (MTAT) was the instrument used for data collection. The validity of the instrument was established by four experts and a pilot study was carried out. The result of the pilot study through test re-test approach Using PPMC statistical tool indicated a reliability index of 0.72. Independent sample t-test was used to analyze the two null hypotheses. The result of the first null hypothesis indicated a significant difference between the two groups in favor of the EG. Moreover, the second null hypothesis shows no significant difference in gender. It was recommended that Van Hiele's instructional model should be incorporated in the curriculum as model of teaching longitude and latitude.
Keyword: mathematical thinking, van Hiele, longitude and latitude

Introduction
Advancement in science and technology requires a great deal and mastery of Mathematics. The aim to achieve technological goal cannot be achieved unless and until Mathematics is given a sufficient time and consideration. Moving to achieve a particular level of technological know-how cannot be restricted by other factors like Mathematics do. As the universe is moving and accepting the pen less system with the aim of making a computer-based society, the role of Mathematics is like an engine to a machine. The importance of Mathematics in Nigeria cannot be overemphasized as it is core subject taught from primary one up to the terminal level of secondary education. It is because of the role of Mathematics to the nation building and in nutshell to the development of science and technology in Nigeria was made a compulsory requirement for admission into any tertiary institutions. Therefore, method, strategy, technique, model, materials and personnel should be sufficiently deployed to the area.

Many researches have been conducted in order to improve teaching and learning Mathematics in general. The researches had pin point the weakness of the curriculum, teaching methodology, the instructional materials, teacher development and professionalism, culture and many more variables. These had also called for application of specific strategies, techniques or models to a specific concept in Mathematics. Even with application of some specific strategies, methods or models to various concepts of Mathematics there's one that best fit for particular concept. These may yield fruitful results. This

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make Lince (2016) to believe that Mathematics improve students' logical thinking, analytical, systematic, creative thinking as well as ability to cooperate. These thoughts can be reared through the use of innovative and modern techniques of teaching and learning.

Mathematical thinking ability is creative thinking (Lince, 2016). Mathematical thinking ability is a cognitive process of analyzing, synthesizing and comprehending the concept or ideas. According to Yulianti, Sumarmo and Kustiana (2020) mathematical critical thinking signifies the ability to be able to clarify an idea or concept with sufficient and genuine reasons. Mathematical thinking ability is important in decision making. It involves rational thinking and passing judgments with a valid conclusion. It helps the students to identify relevant and irrelevant idea or thought so as to make right deduction to pass a reasonable judgment. This ability needs to be cultivated from the basic to make a sufficient utilization of it. It can build in students through the use of innovative teaching techniques. Students taught by traditional teaching method have low mathematical thinking ability (Yulianti, Sumarm & kustiana, 2020). It indicated that the use of innovative models such as Van Hiele's model can improve students' mathematical thinking ability. So also Casing and Roble (2021) indicated that the use of innovative teaching techniques or models improve students' mathematical thinking ability rigorously. Teachers need to imbibe new teaching techniques or models such as Van Hiele's model in order to improve students' mathematical thinking ability. Similarly, the findings of Brojonegoro (2020) indicate that the use of innovative teaching models increase students mathematical critical thinking ability. This Mathematical critical thinking ability involves mental construction of a genuine and unique idea. It also involves the ability to make a justifiable deduction from infinitely unassembled ideas or concepts. Furthermore, Van Hiele's learning model can increase the mathematical thinking ability of students even at the advance level. This was also re-affirmed by Armah, Cofie & Okpoti (2018)

Gender had always been a concern to the researchers. This is because many people believe that Mathematics is male dominated area. It makes the female students lose interest and develop math phobia. The cause of this had been investigated by the concern educationists. Many attached this to poor teaching method, lack of instructional materials, teachers' personality, school environment and other related factors. According Brojonegoro(2020) Mathematical thinking ability is not an innate ability but can be develop through the use of the right and appropriate teaching model. Female students can as well develop mathematical thinking ability as much as their male counterpart or even better if appropriate techniques are employed. Many researches had portrayed the ability of female students in competing with their male counterpart in various concepts of Mathematics. Such researches include Hassan, Abdullah and Ismail (2020), Conolly (2010) and Aliyu and Ibrahim (2017). All these researches indicated the power of innovative teaching strategy or model in neutralizing the gender inequality.

An education practitioner always tries to find out challenges and problem face by education and come up with a possible solution. Van Hiele was not left out as a teacher; his concern was to improve teaching and learning in some of the most difficult areas of Mathematics. A Dutch educator was hit by the difficulty encountered by students in learning geometry. This make him to pursue an applied research and come up with a model that is so simple and precise. The model is a hierarchical and sequential arrangement of the possible steps to follow in teaching and learning geometry. This had help a lot during the time and many researchers applied the model in various location. The result is fruitful. The steps indicate the geometric thinking level of the students. Learning from the basic level to the highest level. At each level the complexity of the model rise to match the cognitive demand of the students. The steps are applied in learning geometry and other related concepts which may involve the use of geometrical figures. The steps are listed below.

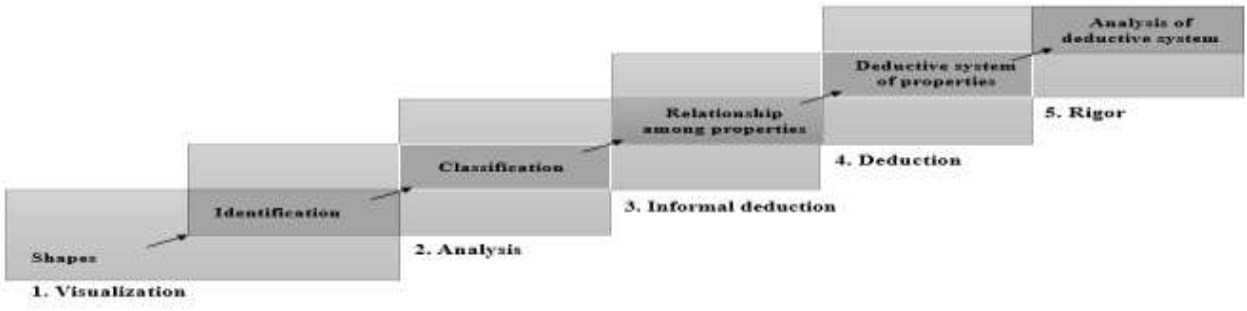

Figure 1: Steps in Geometrical Figures

As human knowledge grows twice in every 24 hours in this 21st century as Ben Carson stated, the knowledge is now beyond recall and retrieved of already stored information. The nation and society in particular require innovation, creativity,

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thinking ability, abstract decision making to solve their immediate problems. This dynamism of nature called for mathematical thinking ability to acquire or access the basic proficiency to explain or find the solution to immediate needs of the society. Thinking is the ability of a person to grasp an idea, concepts or techniques (Lince , 2016). This requires a person to intellectually scrutinize the ideas or concepts with the hope of fishing out the most beneficial component to the society. Creative thinking is mathematical in nature (Lince, 2016). Mathematical thinking is the ability to analyze, synthesize, making generalization, deduction, drawing a conclusion, hitting the ideas from every angle, discuss and be able to solve mathematical problems precisely and concisely with a unique and best method possible (Brojenogoro, 2020). Mathematical thinking may be ability to think out of the box when confronted with a mathematical problem and by extension to a real problem in the society. It is therefore, imperative to develop such Mathematical thinking in the students for fruitful outcome.

Generally, the importance of Mathematics cannot be overemphasized and in particular geometry. Geometrical figures are so useful that we make extensive use of the concepts in our daily activities such as measuring of land and in the building constructions. Such geometric figures play vital role in identifying various location on Earth through the use of longitude and latitude. These concepts become a heart of aviation industry. Clements and Sarama (2011a) in Hassan, Abdullah and Ismail (2020) affirm that geometrical concepts in Mathematics supersedes all other concepts in terms of usefulness. While according to Hassan, Abdullah and Ismail (2020) geometric skills form the bedrock for advancement in science and technology. This shows the immense benefits found in the concepts. It is paramount important that these concepts should be given much attention and consideration in terms of the teaching methodology, model, instructional materials and personnel. Unfortunately, many students find it difficult to understand these concepts as they failed to get a minimum required mark in the external examination such as WAEC and NECO. This issue was identified by WAEC Chief Examiner's report of 2016-2019. Even in 2020 and 2021 the report shows evidence of the lack of understanding geometrical concepts. Some of the remedies to this failure as suggested by chief examiner was teachers to imbibe a new strategy or techniques or models to simplify the concepts in order to improve students' performance. Many efforts were made to remedy this situation such as the improvisation of geometrical figures both wooden and plastic. School and other professional bodies' efforts such as conferences and seminars. But yet the failure persists up to 2021. It is because of this the researchers deemed it necessary to measure the impact of Van Hiele's instructional model on longitude and latitude in order to assess students' mathematical thinking ability.

Objectives of the Study

The objectives of this study are to assess and measure the mathematical thinking ability level of senior secondary school students in Sokoto State through the intervention of Van Hiele's learning model on longitude and latitude concepts. Specifically, the following objectives are addressed in this research.

1. To examine Mathematical thinking ability on longitude and latitude using Van Hiele Model among secondary school students in Sokoto State.
2. To investigate mathematical thinking ability on longitude and latitude using Van Hiele's model between male and female secondary school students in Sokoto State.

Research Questions

The research was guided by the following questions

1. What is mean difference in mathematical thinking ability between students taught longitude and latitude using Van Hiele's model and those expose to traditional model in Sokoto state?
2. Would there be mean difference in the mathematical thinking ability between male and female students taught longitude and latitude using Van Hiele's Model in Sokoto state?

Null Hypotheses

Two null hypotheses were generated and tested at 0.05 level of statistical significance.

1. HO_1: There's no significant difference in the mathematical thinking ability between students taught longitude and latitude using van Hiele's model and those expose to traditional model in Sokoto State.
2. HO_2: There's no significant difference in the mathematical thinking ability between male and female students taught longitude and latitude using Van Hiele's model in Sokoto State.

Methodology

This research aims at investigating mathematical thinking ability on longitude and latitude using Van Hiele's model among secondary school in Sokoto State. Because of the inability of the researchers to make random assignment to the various subjects a quasi-experimental design was employed entailing pretest and posttest. 28,897 students form the population for this

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study. The sample size was determined through the use of Krejcie and Morgan (1970) table of determining sample size. 267 students were selected from two secondary schools used as intact class. The secondary schools were selected using simple random sampling techniques through balloting method. The two schools were randomly allocated to experimental and control group. 125 and 142 students form the control and experimental group respectively. There are 72 male and 53 female students in control group. While in the experimental group there are 83 male and 59 female students. Mathematical Thinking Ability Test (MTAT) was the instrument used to capture the required data. The test undergoes serious scrutiny by four experts and a final draft was drawn and administered to the sample. A portion of the population was tested and retested with the instruments. The result was analyzed using Pearson Product Moment Correlation. An index of 0.72 was found as such the reliability of the instrument is guaranteed. Descriptive statistics was employed in answering research questions. The data collected was from two independent samples as such an independent sample t-test was used to analyze the two stated null hypotheses.

Results

Descriptive statistics such as mean and standard deviation was used to answer research questions one and two. Independent sample t-test was used to analyze null hypotheses one and two.

Research Question 1

What is mean difference in mathematical thinking ability between students taught longitude and latitude using Van Hiele's model and those expose to traditional model in Sokoto state?

Table 1: Mathematical Thinking Ability Mean Score in EG and CG

Group	N	Mean	Std Dev.	Std Error Mean
Experimental	142	3.82	2.44	0.12
Control	125	2.59	0.95	0.09

Field data (2023)

From the table above, the mathematical thinking ability of students when exposed to Van Hiele's learning model was improved. Even though the mean difference is meager but at least the intervention had shown in the mean score

Table 2: Mathematical Thinking Ability Levels of Experimental and Control Group

Group	Mathematical Thinking Ability Level							
	Level 1		Level 2		Level 3		Level 4	
	Passed	Failed	Passed	Failed	Passed	Failed	Passed	Failed
Control	120 (96%)	05 (4%)	50(40%)	75 (60%)	43(34%)	82(66%)	31(25%)	94(75%)
Experimental	139(98%)	03(2%)	114(80%)	28(20%)	101 (71%)	41(29%)	97(68%)	45(32%)

Research Question 2

Would there be mean difference in the mathematical thinking ability between male and female students taught longitude and latitude using Van Hiele's Model in Sokoto state?

Table 3: Mathematical Thinking Ability of Male and Female in the Experimental Group

Gender	N	Mean	Std Dev.	Std Error Mean
Male	83	3.62	1.71	0.12
Female	59	3.20	1.60	0.20

Field data (2023)

The table above indicated that Van Hiele's learning model is gender friendly. Even though there's difference in the mean performance of male and female mathematical thinking ability. The difference is meager not sufficient enough to justify a change due to gender.

Table 4: Mathematical Thinking Ability Level of Male and female Students in EG

Gender	Mathematical Thinking Ability Level of Male and Female

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	Level 1		Level 2		Level 3		Level 4	
	Passed	**Failed**	**Passed**	**Failed**	**Passed**	**Failed**	**Passed**	**Failed**
Male	83	0	80	3	71	12	68	15
Female	56	3	34	24	30	29	29	30

Hypothesis 1

HO_1: There's no significant difference in the mathematical thinking ability between students taught longitude and latitude using van Hiele's model and those expose to traditional model in Sokoto State.

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Table 5: Analysis of Mathematical Thinking Ability of Experimental and Control Group

Group	N	Mean	Std Dev.	P-Value	Decision
Experimental	142	3.82	2.44	0.001	Rejected
Control	125	2.59	0.95		

Decision Criterion: Reject H_o if P≤0.05. Field data (2023)

From the table above the p-value is 0.001. Therefore, the stated null hypothesis is rejected. This indicated there's significant difference in mathematical thinking ability of students taught longitude and latitude using Van Hiele's learning model and those taught using traditional method. The result favors the experimental group.

Hypothesis 2

HO_2: There's no significant difference in the mathematical thinking ability between male and female students taught longitude and latitude using Van Hiele's model in Sokoto State.

Table 6: Analysis of Mathematical Thinking Ability of Male and Female in the Experimental Group

Gender	N	Mean	Std Dev.	P-value	Decision
Male	83	3.62	1.71	0.100	Accepted
Female	59	3.20	1.60		

Decision Criterion: Reject H_o if P≤0.05 Field data (2023)

Table 4 above shows p-value of 0.100 which greater than 0.05. Therefore, the stated null hypothesis is accepted. This indicated no significant difference in the mathematical thinking ability of male and female students when taught longitude and latitude concepts using Van Hiele's learning model. The model is gender friendly.

Discussion of Findings

From the analysis, the first null hypothesis indicated a significant difference in the mathematical thinking ability of students taught longitude and latitude using Van Hiele's learning model and those exposed to traditional method in favor of experimental group. This finding is not void because it further confirms the result submitted by Yulianti, Sumarmo and Kustiana (2020). Van Hiele's learning model is an innovative technique to improve students' mathematical thinking ability in geometrical figures and other related concepts. So also Casing and Roble (2021) submitted a similar finding. They posited that the use of innovative strategy or model improve students' mathematical thinking ability vigorously.

The second null hypothesis indicated no significant difference in mathematical thinking ability of male and female students exposed Van Hiele's learning model. This confirm no gender disparity when Van Hiele's model is used in teaching longitude and latitude. This finding was supported by some recent results such as Aliyu and Ibrahim (2017), Hassan, Abdullah and Ismail (2020) and Conolly (2010). Therefore, Van Hiele's model as an innovative teaching model if properly used and utilized can bridge gender disparity in geometric concept and other related topics such longitude and latitude.

Conclusion

At the terminal of this research, the findings indicated that Van Hiele's instructional model improve students' mathematical thinking ability better than the dominant method used. It further revealed an equal chance for female students to perform and enhance their mathematical thinking ability as well as their male counterpart.

Recommendations

1. Van Hiele's learning model should be used in teaching and learning longitude and latitude
2. The model can also be use in teaching other geometry aspect such as trigonometry.
3. Workshops can further be done in other state by ministries of education or other education stakeholders in order to make the model widely known

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